**Chapter 4. Vector Space**

Vector spaces with real scalars are called ***real vector spaces*** and those with complex scalars are called ***complex vector spaces***. For now, we will be concerned exclusively with real vector spaces.

**4.1 Real Vector Spaces**

Let V be a nonempty set of objects, on which two operations are defined:

1. Addition
2. Multiplication by scalars

With the following properties:

1. If and are elements in V, then is in V. (V is closed under addition)
2. , for all , in V. (holds Commutative Law)
3. (holds Associative Law)
4. There is an object in V, called the zero vector for V such that for each in V. (have Additive Identity)
5. For each in V, there is an object in V, called a negative of , such that . (have Additive Inverse)
6. If is any scalar and is any object in V, then is in V. (Closed under Scalar Multiplication).
7. 1 (have Multiplicative Identity)

then V is called a vector space and the objects in *V are* ***vectors***.

**Example 1:** Let , prove that V is a vector space under the usual operations of addition and scalar multiplication defined by:

**Solution:**

1. V is closed under addition. (as defined)
2. Let
3. Let
4. Let
5. Let then there exist
6. V is closed under scalar multiplication. (as defined).

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As the set V satisfies all the properties, so is vector space.

**Example 2:** Let , prove that V is a vector space under the usual operations of addition and scalar multiplication defined by:

**Example 3:** Let , under the usual operations of addition defined by:

And if k is any scalar number, then define

The addition operation is standard one on , but the scalar multiplication is not.

Check whether V is vector space or not?

**Solution:**

All properties of addition are satisfied. (Check it by yourself)

Let’s check the properties of scalar multiplication.

1. Let in V, then
2. Let

As the 7th property does not satisfied So it’s not a vector space.

**Example 4:**

Check whether V is vector space or not?

The set of all pairs of real numbers of the form . i.e.

with the standard operations on .

**Solution:**



V is closed under addition.



Then

So V is a vector space.

**Example 5:** Check whether V is a vector space or not.

V=the set of all pairs of real numbers of the form , where .i.e.

With standard operations on .

**Solution:**

As

1. Let

Because So,V is closed under addition.

1. (Easy to verify)
2. (Easy to verify)
3. Let
4. Let Then there doesn’t exist because should be positive.

So 5th property fails.

V is not vector space.

**Example 6:** Show that the set of all pairs of real numbers of the form with the operations

&is not a vector space.

**Example 7:** Determine whether the set of all triples of real numbers with standard vector addition but with scalar multiplication defined by

is a vector space or not.

**Example 8:** Determine whether the set of all pairs of real numbers of the form with the operations

is a vector space or not.

**Example 9:** Determine whether V is a vector space or not.

V= the set of all triples of the form with the operations

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**Example 10:** Determine whether V is a vector space or not.

Let V be the set of all matrices with real entries and take the vector space operations on V to be usual operations of matrix addition and scalar multiplication i.e.

**Solution:**

1. V is closed under addition.

Similarly, you can prove all the properties of scalar multiplication. (Prove it by yourself).

So, V is a vector space.

**Example 11:** Let and define operations on V to be the usual operations of addition and scalar multiplication.

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Then V is vector space.